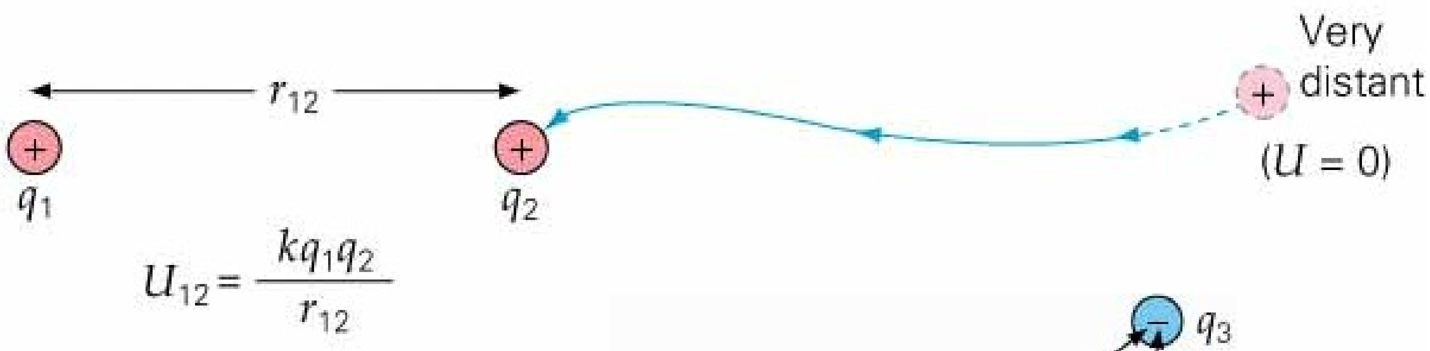
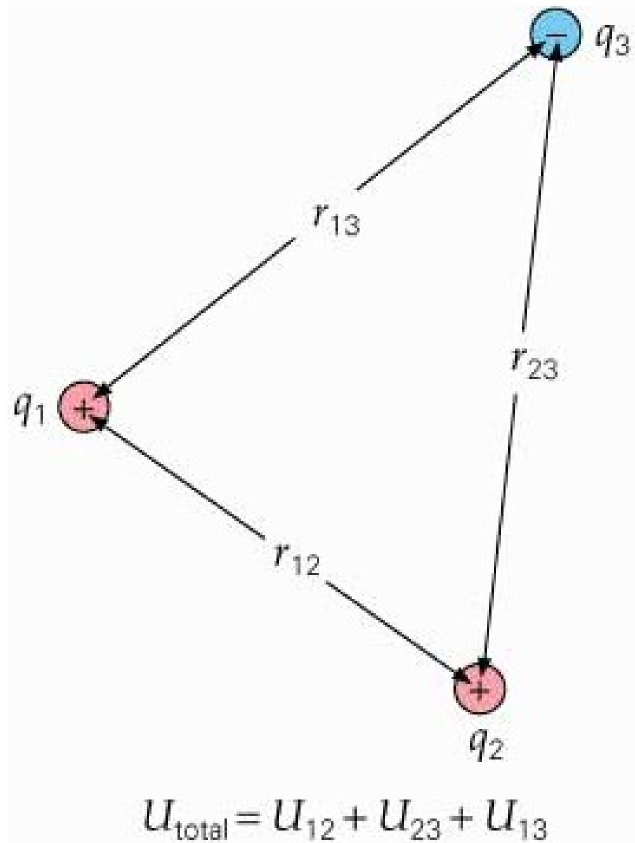
Review

- **Energy** associated with building up net charges



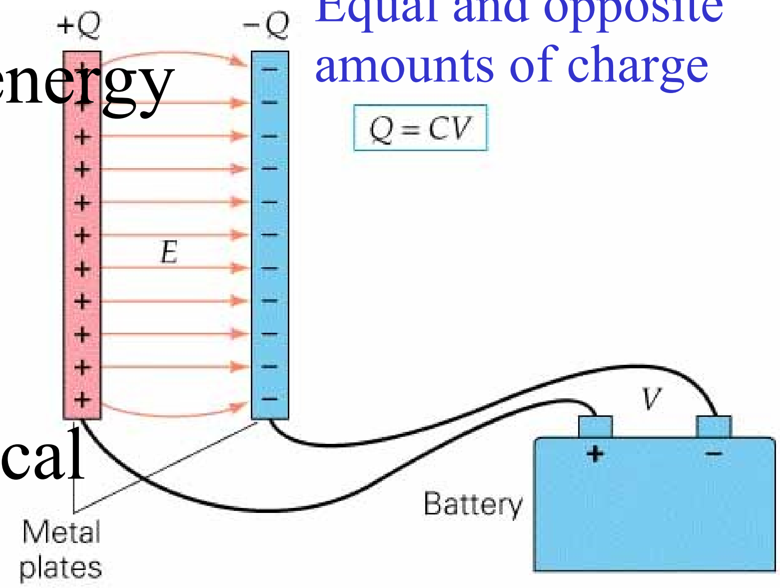
- Three charges



$$U_{ij} = k\frac{q_i q_j}{r_{ij}}$$

Capacitors

- Any charge separation stores energy
- Parallel Plates
  - Most typical capacitor
  - Uniform E field
- Common component in electrical circuits
- **Capacitance: the "charge per volt" on a capacitor**



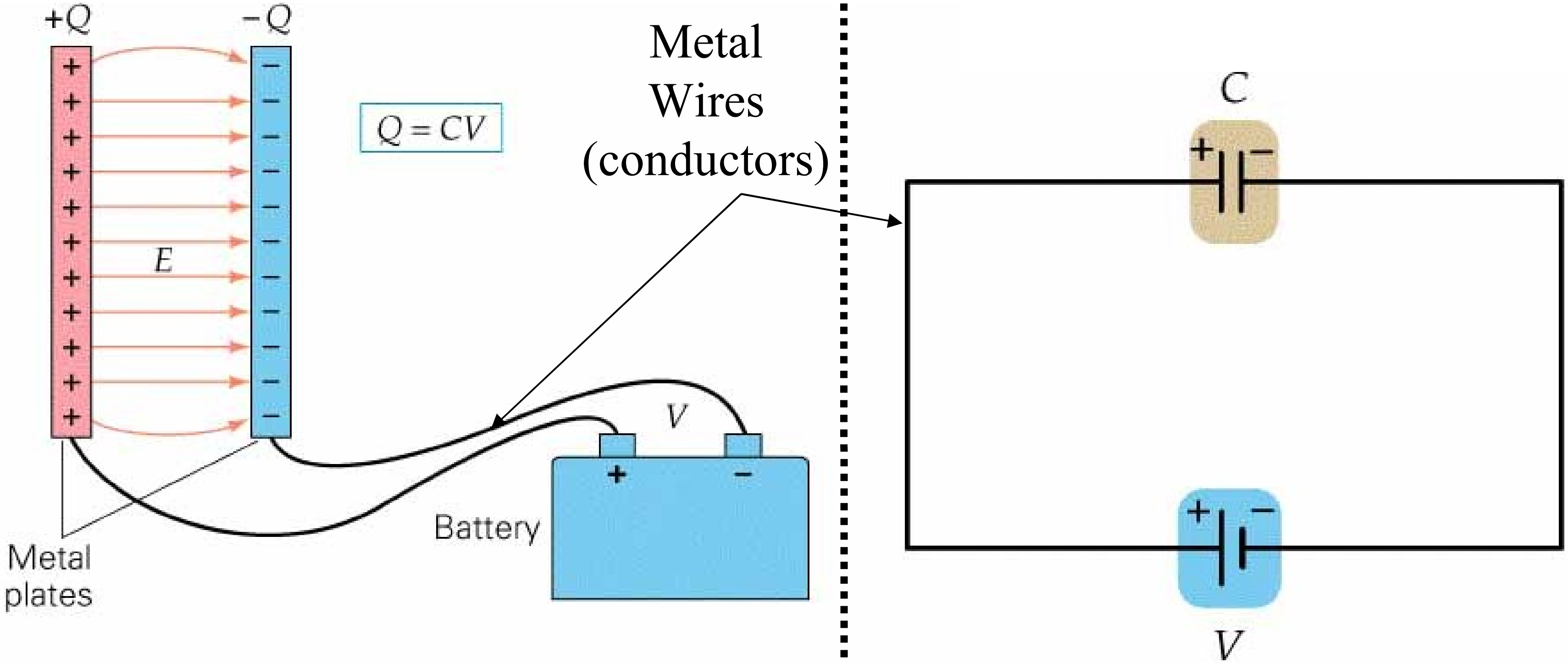
(a) Parallel-plate capacitor

$$C \equiv \frac{Q}{V}$$

Capacitance governs ...
how much charge is required to produce 1 volt on the capacitor (Q=CV),
what the potential difference will be if +-Q of charge is on the plates. (V=Q/C)
Units: 1 Farad = Coulomb/Volt

Notation: $\Delta V = V$ the potential difference between the plates

Capacitors



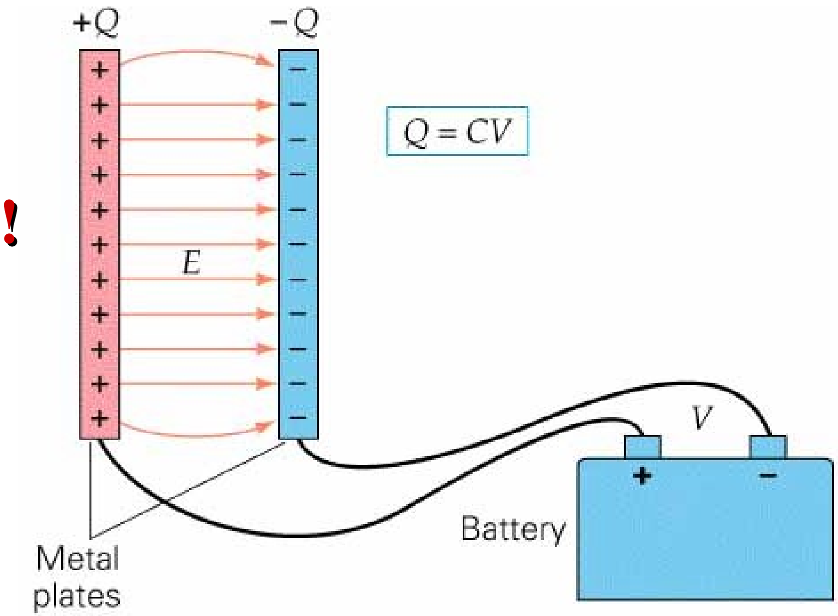
(a) Parallel-plate capacitor

(b) Schematic circuit diagram

Note the difference between capacitors and batteries

Compute Capacitance of P.Plate Capacitor

- Physical Dimensions determine the capacitance
- **Independent of charge and voltage!**



(a) Parallel-plate capacitor

$$E \equiv \frac{4\pi kQ}{A}$$

$$(-)\, V = Ed \equiv \frac{4\pi kQd}{A}$$

$$C = \frac{Q}{V} = \frac{QA}{4\pi kQd} = \left(\frac{1}{4\pi k}\right)\frac{A}{d} = \varepsilon_o \frac{A}{d}$$

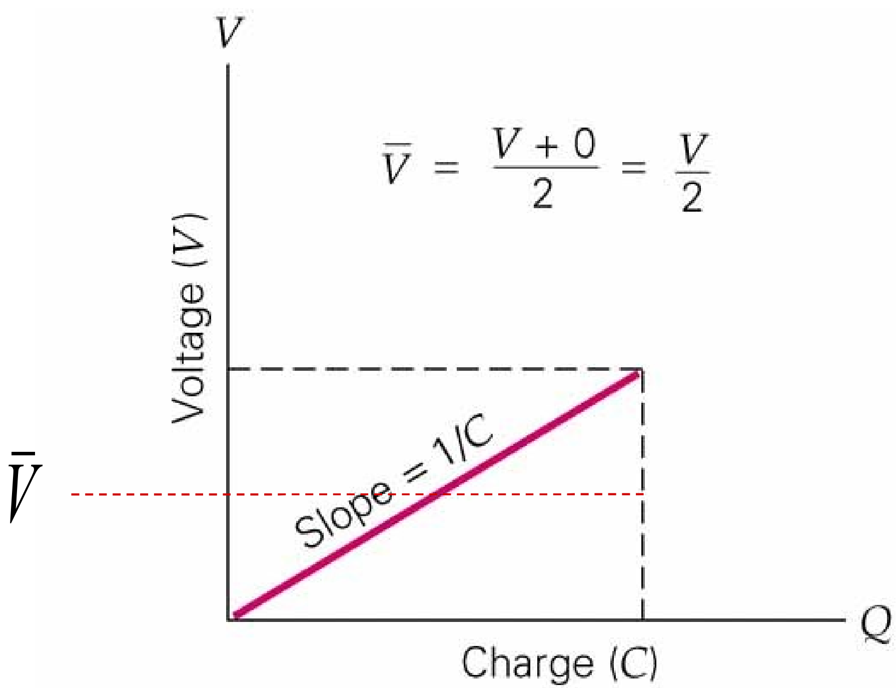
$$\boxed{C = \varepsilon_o \frac{A}{d}}$$

For Parallel Plates only

$$\varepsilon_o = 8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2}$$

Charging a Capacitor

- As charge is added, the potential difference between the plates increases $C \equiv \frac{Q}{V} \Rightarrow V = Q/C$ C is constant (fixed value)
- Stored energy also increases
  - think of point charges $W = U_e = q\Delta V$
  - more difficult to move each additional charge



$$W = U_c = q\bar{V} = \frac{1}{2}QV$$

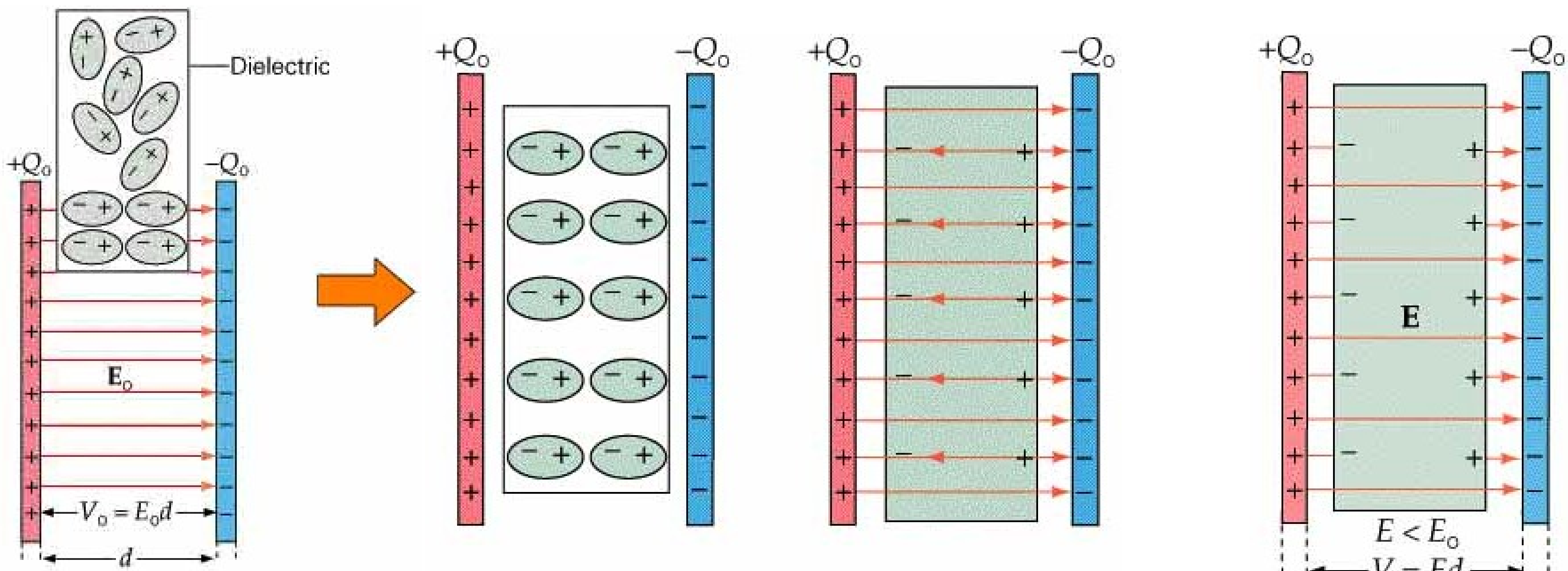
$$C \equiv \frac{Q}{V} \Rightarrow Q = CV$$

$$U_c = \frac{1}{2}QV = \frac{Q^2}{2C} = \frac{1}{2}CV^2$$

Energy of a charged capacitor

Dielectrics

- Capacitor charged and **<u>battery disconnected</u>**
  - Q is constant - nowhere to go!
  - Work done to align dipoles
  - Field of dipoles partially cancels field from plates
  - E field and Voltage both decrease (V = E/d)



Charged capacitor

Charged capacitor with dielectric inserted

Electric field diagram of same

Effect on electric field and voltage

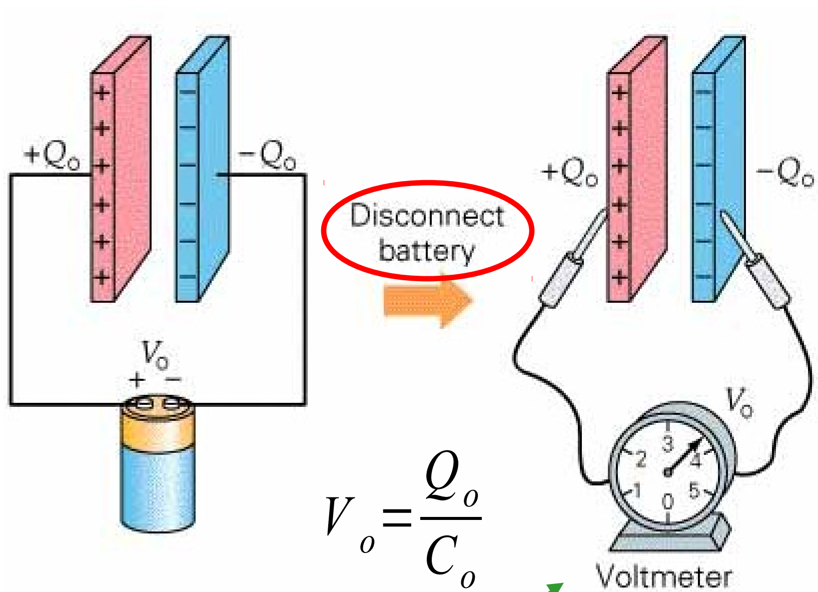
Dielectrics

- **Battery disconnected** $C = \frac{Q}{V}$

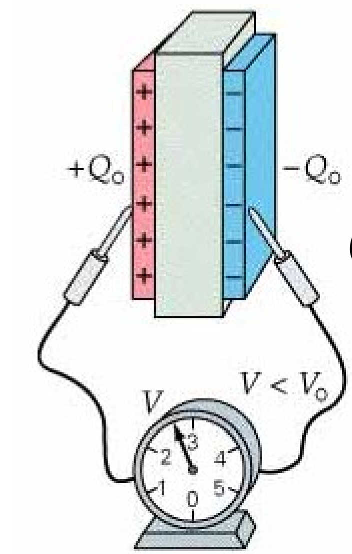
What happens to each quantity?

Charge Q_o is constant

Voltage drops from V_o to V



$$V_o = \frac{Q_o}{C_o}$$



$$C = K\ C_o$$

$$V = \frac{Q_o}{C} = \frac{Q_o}{KC_o} = \frac{1}{K} V_o$$

Voltage decrease

K Dielectric Constant ≥ 1

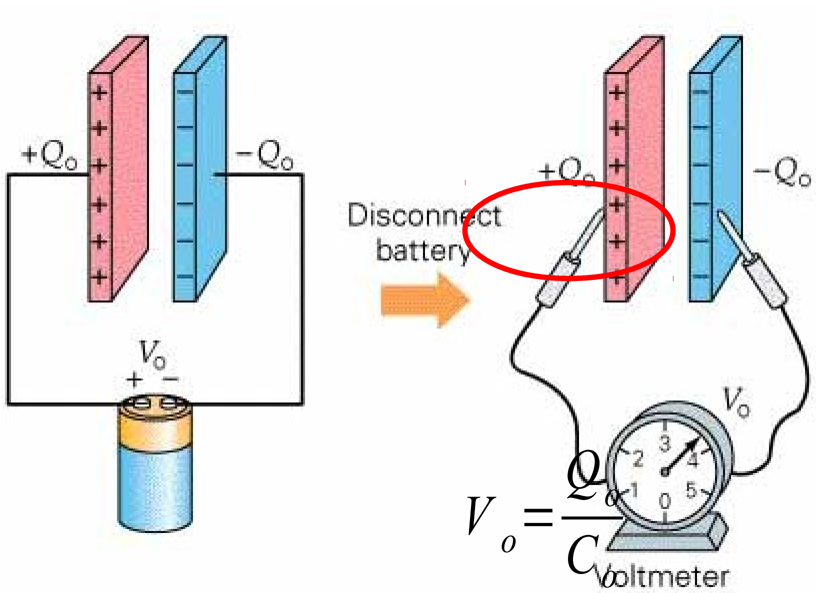
(change in the physical configuration)

VOLTMETER: Measures voltage only – does not hold or supply charge or change the voltage

Dielectrics

- **Battery disconnected**

Charge Q_o is constant

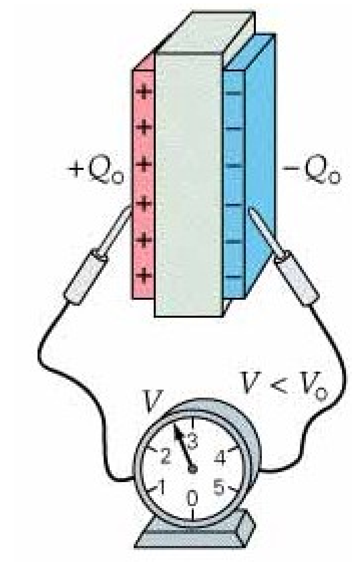


$$V_o = \frac{Q_o}{C_o}$$

Voltage drops from V_o to V

$$V = \frac{1}{K} V_o$$

$$C = K\, C_o$$



$$U_o = \frac{1}{2} Q_o V_o = \frac{{Q_o}^2}{2C_o} = \frac{1}{2} C_o {V_o}^2$$

Energy decreases (work done to align dipoles)

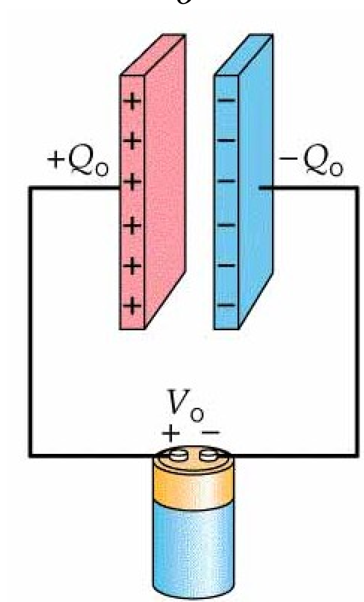
$$U = \frac{1}{2}\, Q_o V = \frac{Q_o^2}{2C} = \frac{1}{2} C V^2$$

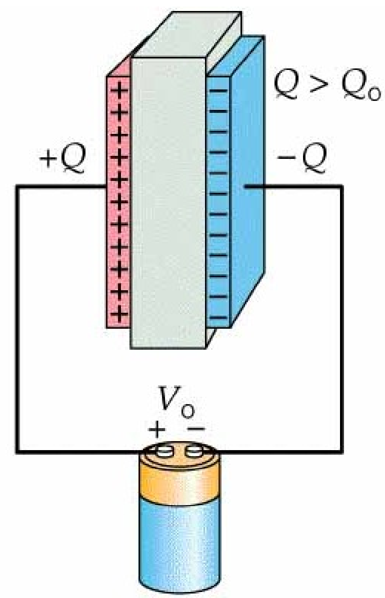
$$U = \frac{1}{2}\, Q_o \frac{V_o}{K} = \frac{Q_o^2}{2\,KC_o} = \frac{1}{2} K C_o \left(\frac{V_o}{K}\right)^2 = \frac{1}{K} U_o$$

Dielectrics

- **<u>Battery connected</u>** $C = K\,C_o$
  - More charge flow out of the battery to *maintain* V_0
    - voltage is fixed by the battery! $Q = K\,Q_o$
  - More energy is stored by the capacitor
    - energy stored in the aligned dipoles

$$U_o = \frac{1}{2} Q_o V_o = \frac{Q_o^2}{2C_o} = \frac{1}{2} C_o V_o^2 \qquad U = \frac{1}{2} Q V_o = \frac{Q^2}{2C} = \frac{1}{2} C V_o^2 = K U_o$$





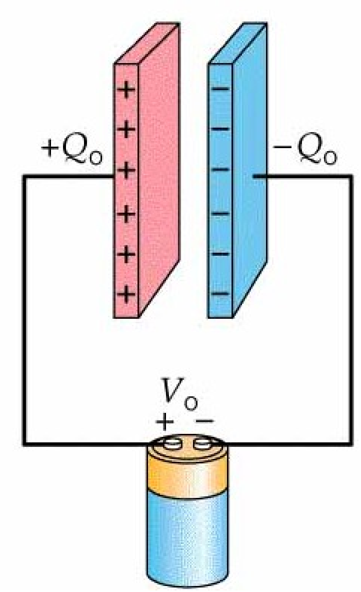
Dielectrics

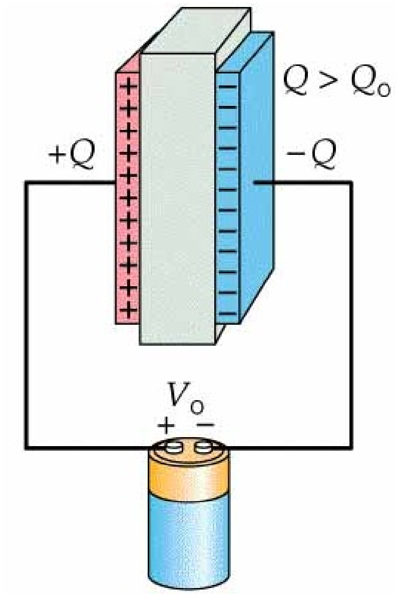
- Parallel Plates

$$C_o = \varepsilon_o \frac{A}{d}$$

$$C = \varepsilon \frac{A}{d} = K\varepsilon_o \frac{A}{d}$$

Dielectric **permittivity** $\varepsilon = K\varepsilon_o$





Energy of a Charged Capacitor

- Three main quantities: *Q, C, V*
  - Along with

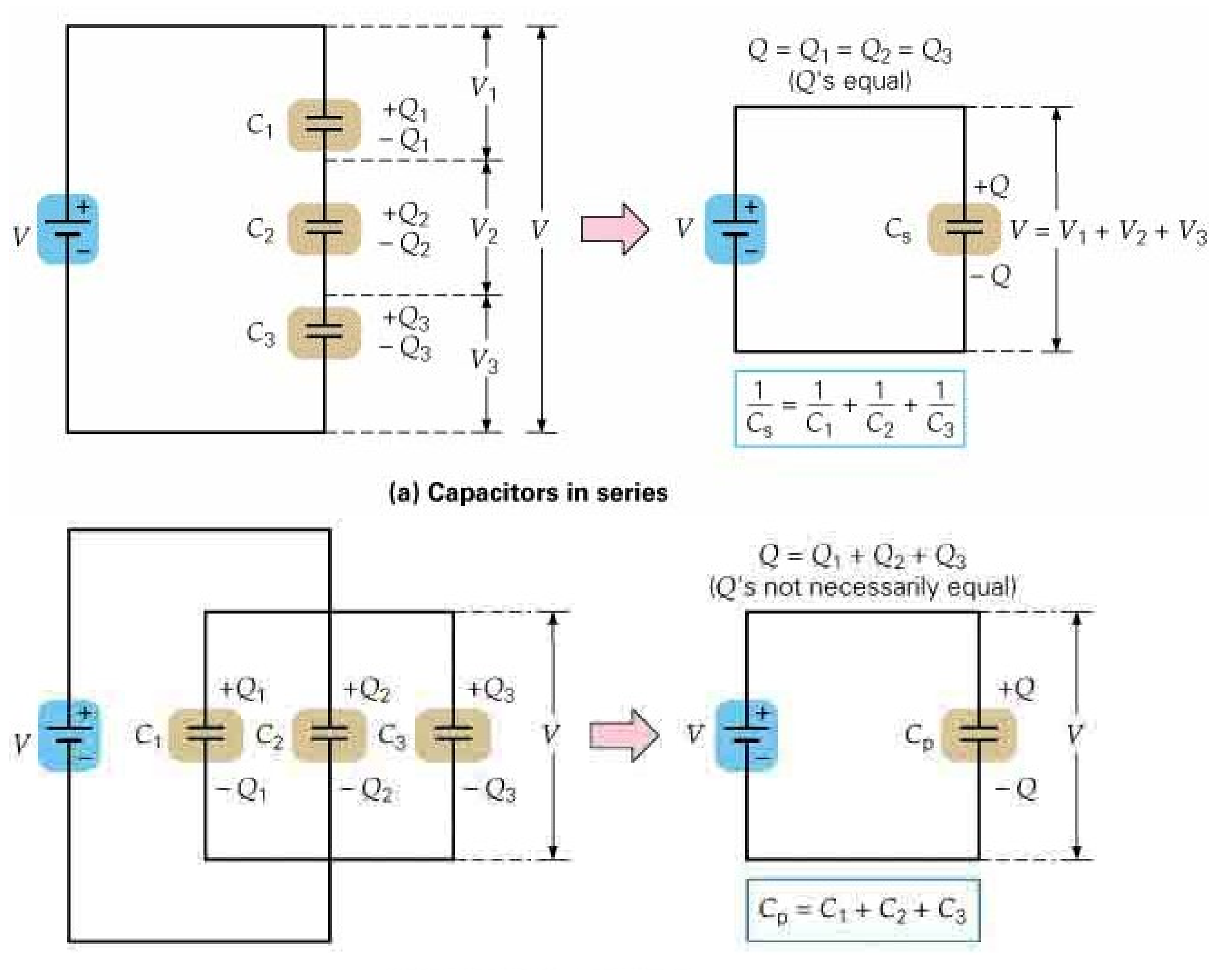
$$C = \varepsilon \frac{A}{d}$$

- Identify what is fixed and what changes
  - Note we can write the energy with any *two* of the three quantities.

$$U_c = \frac{1}{2} QV = \frac{Q^2}{2C} = \frac{1}{2} CV^2$$

Combining Capacitors

- Two fundamental arrangements
- Goal to combine them into one *equivalent capacitance*



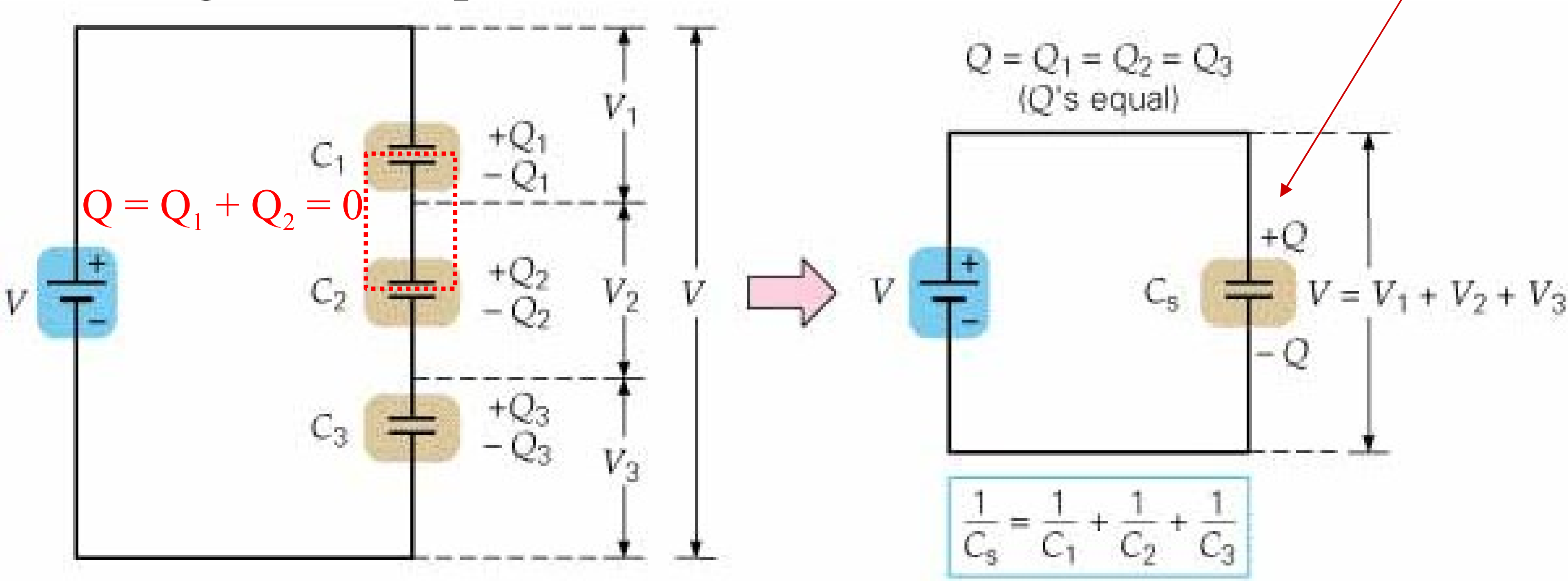
(a) Capacitors in series

(b) Capacitors in parallel

Capacitors in Series

- Charge in each plate is the same

Equivalent capacitor



(a) Capacitors in series

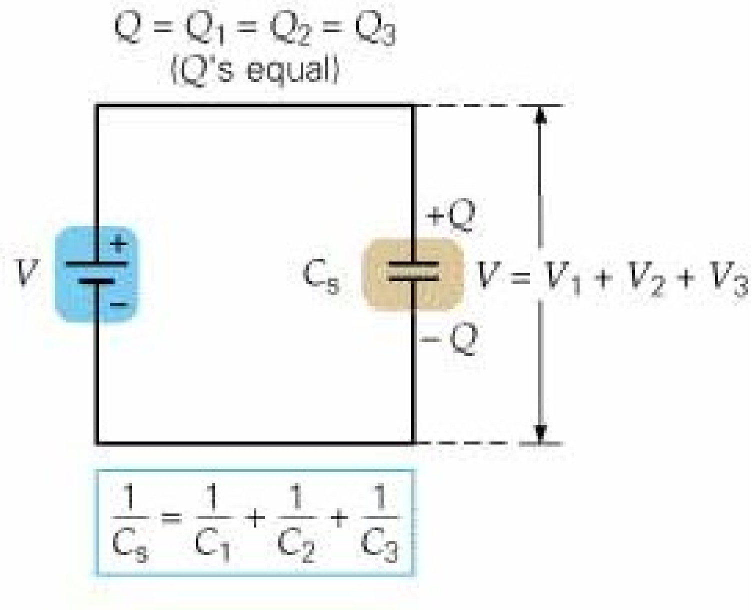
$$C_1 = \frac{Q}{V_1} \quad C_2 = \frac{Q}{V_2} \quad C_3 = \frac{Q}{V_3}$$

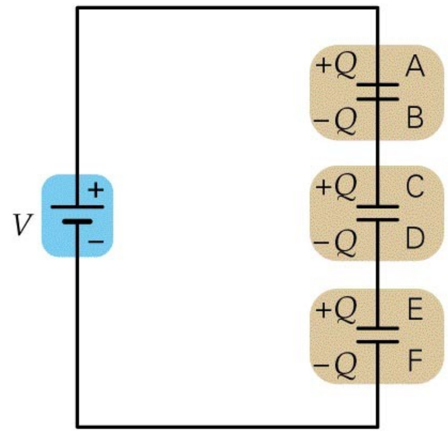
$$V_1 = \frac{Q}{C_1} \quad V_2 = \frac{Q}{C_2} \quad V_3 = \frac{Q}{C_3}$$

$$V = V_1 + V_2 + V_3 \quad V = \frac{Q}{C_s}$$

Capacitors in Series

- Usually get C_s to get Q, then then figure out the V's





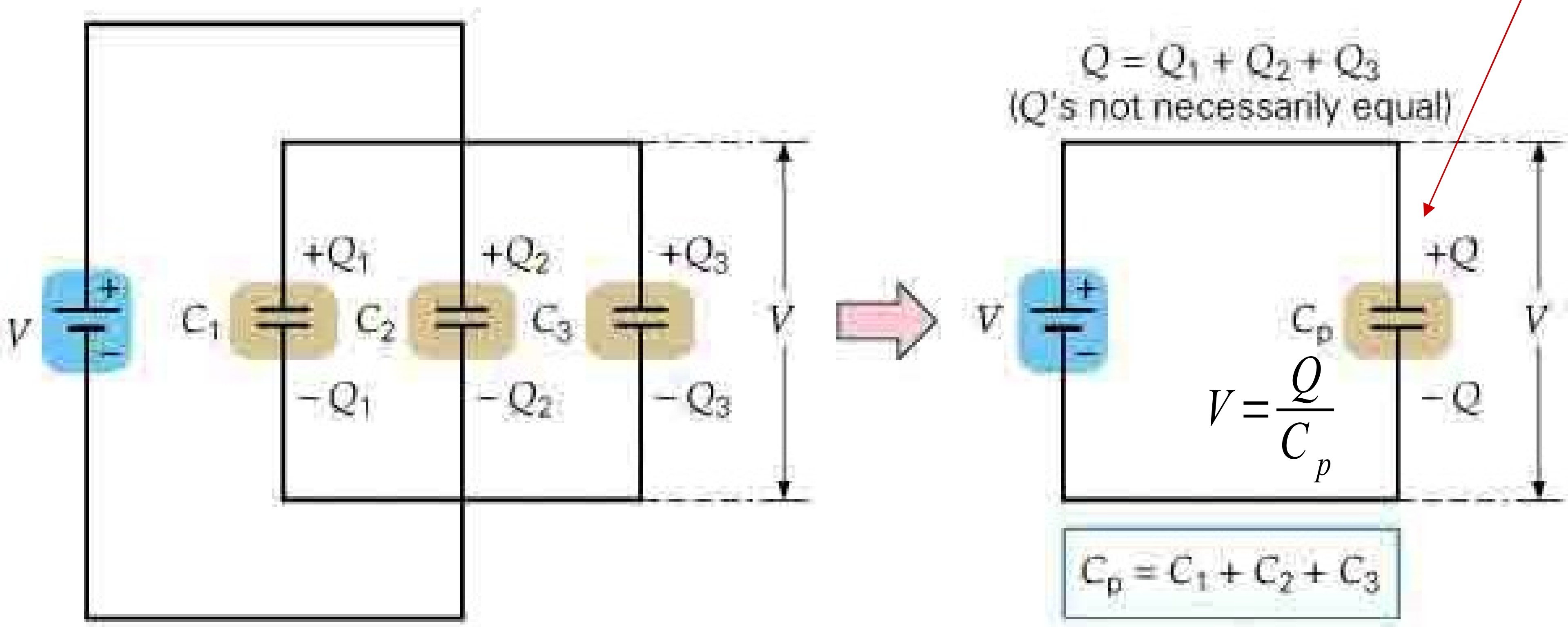
- Now expand the circuit back out

$$V_1 = \frac{Q}{C_1} \quad V_2 = \frac{Q}{C_2} \quad V_3 = \frac{Q}{C_3}$$

Capacitors in Parallel

- Voltage on each plate is the same $V = V_1 = V_2 = V_3$

Equivalent capacitor



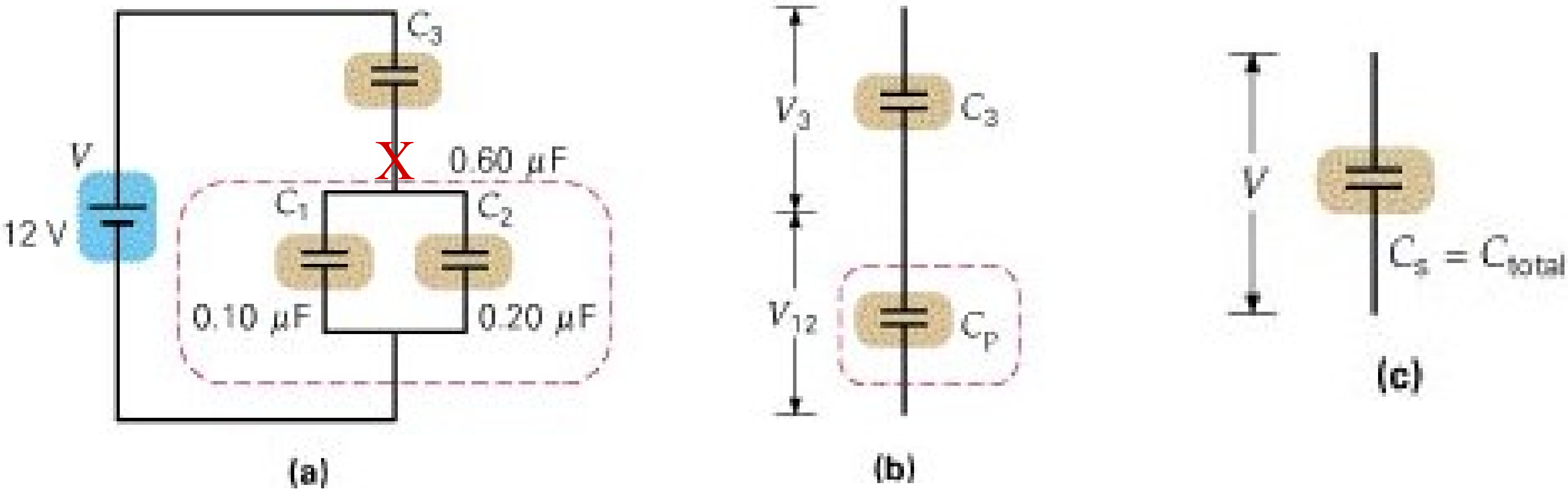
(b) Capacitors in parallel

$$Q = Q_1 + Q_2 + Q_3$$

$$Q_1 = C_1 V \quad Q_2 = C_2 V \quad Q_3 = C_3 V$$

Combinations of Series and Parallel

- Get to one equivalent capacitor



First combine C_1 and C_2 that are in parallel

Second combine C_{12} (C_p in diagram) and C_3 in series

CANNOT DO THE FOLLOWING:

C_3 and C_1 in series, then C_2

C_3 and C_2 in series, then C_1

If you did, where would the point marked "X" appear?